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New Higher-Order Ballistic Wind Compensation Method

James Nohl Churchyard*
Brunswick Corporation, Costa Mesa, Calif.

A new synthesis of methods for the calculation of sounding rocket launch angles to compensate for ballistic winds is presented. The launch and burnout angles are transformed from the normal elevation and azimuth angle system to a system that provides nearly linear response of burnout angles to variations in launch angles and ballistic wind components. A regular pattern of perturbations is applied to the nominal trajectory to yield variations in the burnout angles. Two second-order, four independent variable, curve fit polynomials are determined to represent the burnout angles. These polynomials are then solved to yield launch angles as functions of ballistic wind components. The launch angle solution computations are suitable for small minicomputers. The techniques discussed here have been successfully used to support Athena rocket launches at Wallops Island.

Nomenclature

A = 14-element vector of the polynomial coefficients
e = sum square polynomial curve fitting error = ΣE_i^2
E_i = curve fit error = $f_i - F_i$
f_i = data values generated from the polynomial corresponding to F_i
F_i = data points to be fitted by a polynomial which is optimal in a least-squares error sense
G = gradient of the sum square curve fitting error with respect to the polynomial coefficients
H = Jacobian (matrix of second order partial derivatives) of the sum square curve fitting error with respect to the polynomial coefficients
W_x = wind component, positive from the tail
W_y = wind component, positive from left side
x,y,z = coördinte system axes, with different sets distinguished by subscripts: g for local geodetic tangent north-east-down; b for body roll-pitch-yaw axes; d for displaced body axes
γ = burnout velocity vector horizontal flight path angle, same sense as θ
ζ = relative yaw angle at launch
Z = relative velocity yaw angle at burnout
θ = launcher pitch angle, positive upward from local geodetic tangent plane
ξ = relative pitch angle at launch
Ξ = relative velocity vertical plane angle at burnout
σ = burnout velocity vector vertical flight path angle, same sense as ψ
ψ = launcher azimuth angle, positive clockwise from north to east in local geodetic tangent plane

Subscripts

d = displaced from the nominal value
n = nominal value

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*Assistant Principal Engineer, Engineering Dept., Defense Div. Associate Fellow AIAA.

Introduction

THE Athena rocket family consists of several distinct types of vehicles. The number of major motor stages varies from two to four. All-up weight varies from approximately 16,000 pounds to 31,000 pounds. Except for a few initial test vehicles, each vehicle performs two separate functions—boost and re-entry. The boost stages loft the final stages to altitude. The upper stages are then reoriented outside the atmosphere and are used to drive the payload into IRBM to ICBM re-entry conditions by burning while pointed downward. A midcourse radio command system is used to update the final stage's burning attitude and ignition time to remove the accumulated effect of booster errors on the payload re-entry and impact conditions.

During the boost phase of flight the Athena is uncontrolled but statically stable. Hence it weathercocks into the wind and veers off course. The magnitude of the error induced by the wind can be readily determined by simulation for a given wind field. Because the effects of wind cause both booster impact location errors and payload re-entry performance errors, the wind effects must be minimized. For a given criterion of trajectory acceptability there usually exist launcher settings which compensate for the wind field and bring the compensated trajectory near the nominal no-wind trajectory.

A variety of techniques have been used to provide wind compensated launcher angles for the Athena. Table 1 shows the significant events in the history of the Athena program with special emphasis on the wind compensation method. Basically the method used for wind compensation for the first 152 Athenas consisted of iterative adjustments of launcher settings verified by a five-degree-of-freedom (5-DOF) trajectory simulation through the measured wind field. For the first 114 vehicles the criterion used for an acceptable trajectory was the nearness of the compensated booster impact position to the nominal impact position. The trajectory simulation employed used a considerable number of simplifying assumptions to reduce the necessary computer time. Toward the end of the series of launches at White Sands Missile Range (WSMR), New Mexico, tables of adjustments were employed to the 5-DOF trajectory results to improve the accuracy of the compensating launcher angles. Starting with flight number 115 the iteration method was altered to match booster burnout angles instead of booster impact point coordinate. This change was desirable because the vehicle was being used near its maximum range point where impact constraint solutions were not always possible. That is, the maximum range capability is a function of the wind field. And for some winds it may not be possible to achieve a nominal maximum range impact point. Also, matching

Table 1 Athena wind compensation historical summary

Flight no.	Date	Remarks
1	2/10/64	Iterated 5-DOF trajectory solution for impact position; fixed destruct corridor
13	6/21/65	First use of floating destruct corridor
115	6/5/69	First use of burnout angle iteration criterion
141	8/24/73	Final Athena launched at WSMR
142	12/12/73	First Athena launched at Wallops Island using launcher setting table from 5-DOF runs
144	2/13/74	First Athena launched at Wake Island using linear $\theta_1 - \theta_2$ compensation
152	6/23/74	Last Athena launched at Wake Island
158	12/15/76	First Athena launched at Wallops using ζ-ξ second-order method
159	3/20/77	Most recent launch of Athena at Wallops

Totals	
WSMR	141
Wake	7
Wallops	11
	159

burnout angles reduces payload re-entry dispersion significantly, but does not materially increase booster impact dispersions.

Initially a corridor in velocity space about the nominal azimuth was used for the destruct decision by the Missile Flight Safety Officer. The width of this corridor was proper (about ±4.5 deg), but a criterion was adopted that the vehicle, regardless of the magnitude of the offset in launch azimuth, should be back inside the nominal corridor within a fixed time. This turned out to be an invalid criterion, especially with a definite effect due to the lowered jet stream in the winter months. So a variable corridor was adopted which was based on a trajectory solution acquired late in the countdown. This floating corridor provided a far better means for assessing flight safety factors since the actual wind profile was incorporated in the expected corridor shape.

In 1973 the last Athena was launched at WSMR, the first was launched at Wallops Island, and plans were being finalized to launch a series from Wake Island, about 2000 miles west of Hawaii. Wake Island had no meteorological support facilities other than a small National Oceanic and Atmospheric Administration (NOAA) station. The Wallops facility could support the launch activity with a computer up until about T-20 minutes, but the simulation of trajectories during the countdown was not feasible. So new methods had to be proposed, tested, and implemented. For the Wallops application it was decided to use the 5-DOF program to generate tables of iterated launcher solutions for a matrix of winds. The accuracy of these results seemed to be compatible with the program objectives.

For the Wake Island application far more stringent requirements existed. Here both safety and payload requirements dictated more accuracy in the wind measurement and compensation system. One method that was proposed, evaluated, and found unsatisfactory was to generate polynomial curve fits to iterated solution launcher settings. In order to accurately model the launcher azimuth angle response, a fourth-order curve fit using the two components of ballistic wind was attempted. However, the pattern of points chosen did not provide enough information to define the necessary coefficients, so an alternative method[1] proposed by WSMR personnel was adopted. This method used iterated trajectory solutions for eight points of a matrix. The resulting launcher angles were transformed to $\theta_1 - \theta_2$ angles which are launcher declination angles in the east-vertical plane and the south-vertical plane. Linear coefficients which yield launcher correlations as functions of ballistic winds were then computed for the $\theta_1 - \theta_2$ angles. Different coefficients were used for headwind and tailwind components. The corrected $\theta_1 - \theta_2$ angles were transformed back to launcher elevation and azimuth angles.

Definition of the $\zeta - \xi$ Angles

The 6-DOF program[2] which gives the most accurate trajectory model was modified to iteratively adjust the launcher angles so that the calculated burnout angles with winds acting on the vehicle matched the nominal values. The iteration variables were chosen to be the $\zeta - \xi$ angles relative to the first iteration launch attitude. The ζ angle is, for a small pitch angle, equivalent to a change in the azimuth angle. The ξ angle is equivalent to a change in the pitch angle. By using these two angles (whose nominal values are both zero) the azimuth singularity is avoided. The singularity would be approached only after a 90 deg change in the pitch angle. This is far outside the expected range of launch angles. The burnout angles were also transformed into a similar system about the nominal burnout velocity orientation. The response of the burnout $Z - \Xi$ angles with launch $\zeta - \xi$ angles is quite linear and nearly uncoupled.

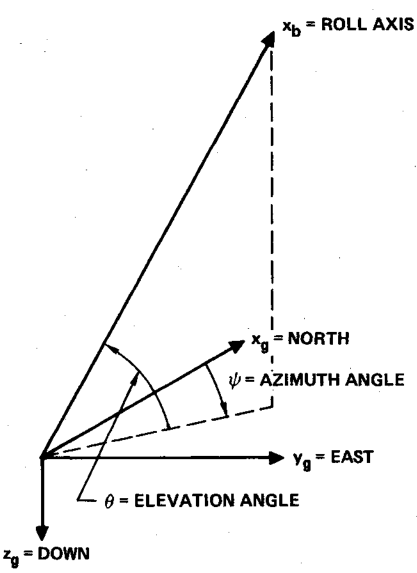

Fig. 1 Relationship between local geodetic horizontal and body axes systems.

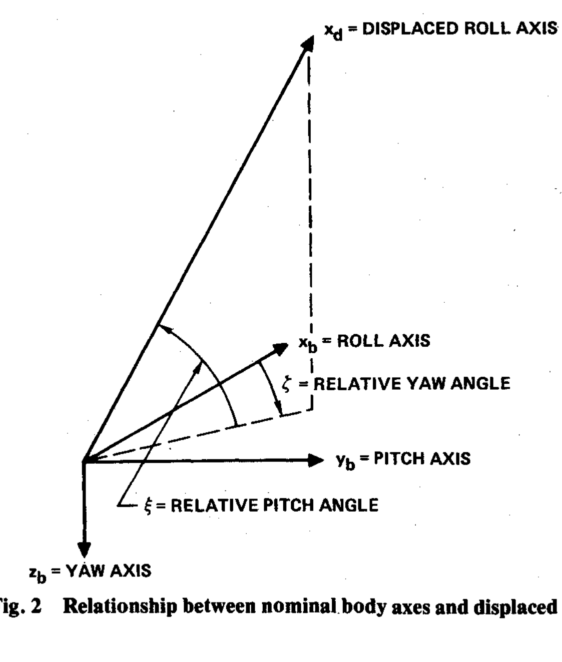

Fig. 2 Relationship between nominal body axes and displaced axes.

Figure 1 shows the relationship between the local geodetic tangent plane Cartesian system and the rocket body coordinate system. The yaw through an angle ψ and then pitch through an angle θ sequence is used to traverse from the local system to the body system. The roll angle is considered to be zero at launch.

Figure 2 shows the relationship between the nominal vehicle axis system and the displaced roll axis. The ζ angle is the relative yaw motion, and is taken first about the nominal yaw axis. Then the ξ angle is a relative pitch angle.

The relationship between the $\zeta-\xi$ angles and the $\theta-\psi$ angles for any displaced launcher orientation are given by the following matrix equation:

$$\begin{bmatrix} \cos\zeta\cos\xi \\ \sin\zeta\cos\xi \\ -\sin\xi \end{bmatrix} = \begin{bmatrix} \cos\theta_n\cos\psi_n & \cos\theta_n\sin\psi_n & -\sin\theta_n \\ -\sin\psi_n & \cos\psi_n & 0 \\ \sin\theta_n\cos\psi_n & \sin\theta_n\sin\psi_n & \cos\theta_n \end{bmatrix} \begin{bmatrix} \cos\theta_d \quad \cos\psi_d \\ \cos\theta_d \quad \sin\psi_d \\ -\sin\theta_d \end{bmatrix} \tag{1}$$

Since the square matrix is orthonormal its inverse is its transpose. So the direction cosines of the displaced vehicle roll axis can be transformed readily between the local geodetic system and the nominal body axes system. Given the direction cosines in the desired system the angles are calculated by inverse trigonometric functions of the direction cosine elements.

Typically the partial derivatives of burnout Z and Ξ with respect to launch angles ζ and ξ are 3.2, with the cross partials being about ± 0.1. Good convergence at even very high launch elevation angles was obtained by using the transformed angles as the iteration variables.

In preparation for the second campaign of Athena launches at Wallops, the experience acquired during the short Wake Island campaign was reviewed extensively. One novel requirement for the Wallops application was that the no-wind launch aximuth angle was to be selected fairly late in the countdown. This implied that the wind compensation method could not be referenced to fixed directions such as in the $\theta_1-\theta_2$ method.

During the Wake Island launch campaign a great many test cases and solutions had been generated and tested against full 6-DOF solutions, some with the actual wind fields and some with ballistic winds. These data were analyzed to determine whether linear or second-order polynomials in the wind components could be fit to the data to yield accurate launcher settings by using the $\zeta-\xi$ angles as the basic variables instead of elevation and azimuth angles. This comparison was performed over 21 cases of wind data from four separate vehicles launched at Wake Island. The mean and standard deviations of the differences between iterated 6-DOF trajectory and the wind compensation estimates of the launch elevation and azimuth angles are shown in Table 2. These results all include the errors due to the reduction of the wind field to a uniform ballistic wind. As can be seen from this table, the $\zeta-\xi$ results have slightly larger mean errors, but smaller standard deviations than the $\theta_1-\theta_2$ results. This closeness is not particularly startling, since both methods use a transformation to move the angular singularity far away from the nominal launcher attitude vector.

Table 2 Launcher setting estimation errors

	Azimuth errors	
	mean	1σ
$\theta_1-\theta_2$ method	−0.0217	0.4069
$\zeta-\xi$ method	0.1254	0.2984
	Elevation errors	
	mean	1σ
$\theta_1-\theta_2$ method	0.0497	0.1059
$\zeta-\xi$ method	0.0579	0.0820

Determining the Evaluation Points

The transformation of the angular basis for describing the launcher attitude is only one aspect of the new ballistic wind compensation method. The second aspect consists in the generation of polynomials that describe burnout angle displacements as functions of variations in the launcher attitude angles and wind components. The use of iterated trajectories is avoided for three reasons:

1) The finite size of the iteration convergence tolerance is a source of noise in the data.

2) The cost of three or four iterative passes per wind condition exceeds that of the noniterative trajectories required.

3) Test case results can be incorporated readily along with the original results.

The basic problem in applying this method is the selection of the conditions of wind and launch angle perturbations to be run. The following discussion outlines a method of ensuring that a selected pattern is adequate.

The discussion is confined to a second order curve fit to data points arranged in a regular pattern. This problem requires the determination of the coefficients for the polynomial. The number of coefficients depends on the number of the independent variables as shown below:

One independent variable

$$f=a_1x+a_2x^2 \tag{2a}$$

Two independent variables

$$f=a_1x+a_2y+a_3x^2+a_4xy+a_5y^2 \tag{2b}$$

Three independent variables

$$f=a_1x+a_2y+a_3z+a_4x^2+a_5xy+a_6xz+a_7y^2+a_8yz+a_9z^2 \tag{2c}$$

Four independent variables

$$f=a_1v+a_2w+a_3x+a_4y+a_5v^2+a_6vw+a_7vx+a_8vy$$
$$+a_9w^2+a_{10}wx+a_{11}wy+a_{12}x^2+a_{13}xy+a_{14}y^2 \tag{2d}$$

In each case the function is about a nominal value, so the constant term is zero. To evaluate the coefficients of the polynomial we must have at least as many data points as there are coefficients. More than this minimum number of data points may be used by using a least-squares fit to optimize the polynomial coefficients.

The number of points which form a regular pattern is a function of the dimensions of the space. In a plane regular polygons exist with any number of points greater than 3. In three-dimensional space only five regular patterns exist. There are the tetrahedron (4 points), the cube (8 points), the octahedron (6 points), the dodecahedron (20 points), and the icasahedron (12 points). Since at least 9 points are required, the icosahedron and the dodecahedron are the only candidates.

In four-dimensional space six regular hypersolids exist.[3] These are named by the number three-dimensional solids that compose the hypersolid. The 5-hedroid is composed of 5 tetrahedra and has 5 vertices. The 8 hedroid is composed of 8 cubes and has 16 vertices. The 24-hedroid is composed of 24 tetrahedra and has 24 vertices. The 120-hedroid consists of 120 dodecahedra and has 600 vertices. The 600-hedroid consists of 600 tetrahedra and has 120 vertices. The last two hypersolids obviously possess far more points than necessary. The 5-hedroid has far too few points. The 8-hedroid, commonly called the hypercube, appears to be ideal. But, when tested, it provides an inadequate basis for the curve-fitting process.

The data points are used to find the polynomial coefficients which meet a least-squares optimality criterion so more than the minimum number may be used. The necessary number of points can be evaluated independently of the actual curve-fitting data. This derivation follows. Suppose we wish to determine the polynomial coefficients from a set of data points F_i, where F is a vector function which we have evaluated at specific points X_i. The error between the known data point and the polynomial approximation is

$$E_i = F_i - f(X_i) \tag{3a}$$

where f is a polynomial as previously written out.

The coefficients must minimize the sum-square error for the least-squares fit:

$$e = \Sigma E_i^2 \tag{3b}$$

By definition, the minimum is that point where the gradient of the total error e is zero. This gradient is taken with respect to the coefficients of the polynomial. For small changes in the value of these coefficients about the minimum the Taylor series expansion is valid:

$$e = \Delta e - e_{\min} = GA + \tfrac{1}{2}A^T HA \tag{3c}$$

Since the sum-square error is second order in the polynomial coefficients, the Jacobian matrix is not a function of the polynomial coefficients. It is solely a function of X_i, the points at which the nonlinear function is evaluated. By definition, H is the gradient of G, so that for any vector of coefficients:

$$G' = G + HA \tag{3d}$$

or, for the desired case where $G' = 0$,

$$A = -H^{-1}G \tag{3e}$$

The value of G does depend on the polynomial coefficients, but the value of H does not. Hence, given the sample points and an assumed value of the coefficients (e.g., all zero) the gradient G and the Jacobian H can be determined and the actual values of the polynomial coefficients for the least squares fit found. This is not an iterative process since the system is exactly first order in the unknown polynomial coefficients.

The Jacobian H depends only on the independent variable data points X_i and not on the function points F_i. The inverse of this Jacobian must exist in order to solve for the polynomial coefficients, which means that H must have a determinant different from zero. Hence, a proposed pattern of data points can be evaluated by calculating the determinant of H before making any trajectory runs.

This calculation has been performed for the two-, three-, and four-variable cases. The results are that 5, 12, and 24 points are needed in order to solve for the polynomial coefficients using a regular sampling pattern. The hypercube, with its 16 vertices, produces a singular matrix for the four-dimensional case, while the 24-hedroid pattern is acceptable. The determinant of the Jacobian matrix for a 24-hedroid inscribed in a unit hypersphere is 62,208. This ensures that the coefficients of the polynomials can be evaluated from the results of the 24 runs.

Solution of Polynomials for Launcher Settings

The polynomial solution method for wind compensation is outlined briefly and a simple pitch plane example discussed. The extension to the complete problem is then presented.

Consider a sounding rocket whose motion is constrained to lie in the vertical plane only, and a wind field which is also constrained to the same plane. The problem here is to determine the change required in the launch pitch angle to bring the burnout flight path angle back to the nominal value in the presence of a wind field. This is a two-variable problem, the independent variables being launch pitch angle and in-plane wind.

The first step is to evaluate the burnout angles at five points of the independent variables. For example:

$$\gamma_1 = f(W\cos 0,\quad \theta\sin 0) - \gamma_n \tag{4a}$$

$$\gamma_2 = f(W\cos 72,\quad \theta\sin 72) - \gamma_n \tag{4b}$$

$$\gamma_3 = f(W\cos 144,\quad \theta\sin 144) - \gamma_n \tag{4c}$$

$$\gamma_4 = f(W\cos 216,\quad \theta\sin 216) - \gamma_n \tag{4d}$$

$$\gamma_5 = f(W\cos 288,\quad \theta\sin 288) - \gamma_n \tag{4e}$$

where W and θ are wind and pitch angle changes which produce comparable effects on the burnout angle. The variable γ_n is the nominal burnout angle. These five data points can be used to determine the coefficients of the polynomial as outlined in the previous section:

$$\gamma = a_1 W + a_2\theta + a_3 W^2 + a_4 W\theta + a_5\theta^2 \tag{5a}$$

For a specific ballistic wind speed the correcting launch angle can be computed from the requirement that the change in the burnout flight path angle be zero:

$$0 = a_1 W + a_2\theta + a_3 W^2 + a_4 W\theta + a_5\theta^2 \tag{5b}$$

The procedure is an iterative one. We start with

$$\theta_0 = -(a_1 W + a_3 W^2)/(a_2 + a_4 W) \tag{5c}$$

Then the iterative formula

$$\theta_1 = -(a_1 W + a_3 W^2)/(a_2 + a_4 W + a_5\theta_0) \tag{5d}$$

is applied until the process converges to the desired value of θ. This is a simple algebraic iteration and does not involve any trajectory runs.

Should there be a requriement to not fly the nominal trajectory, this can be readily implemented by requiring a nonzero burnout angle change.

The extension to the complete problem is based on two four-variable polynomials. One polynomial gives the burnout Z angle change as a function of range and cross-range wind, launcher ζ angle, and launcher ξ angle. The second polynomial gives the burnout Ξ angle change due to the same four variables given by a and b coefficients, then the iterative solution for the launch angles are given by

$$\begin{Bmatrix} \zeta \\ \xi \end{Bmatrix} = -A^{-1}\begin{Bmatrix} a_1 V + a_2 W + a_5 V^2 + a_6 VW + a_9 W^2 \\ b_1 V + b_2 W + b_5 V^2 + b_6 VW + b_9 W^2 \end{Bmatrix} \tag{6a}$$

where

$$A_{11} = a_3 + a_7 V + a_{10} W + a_{12}\zeta + 0.5 a_{13}\xi \quad (6b)$$

$$A_{12} = a_4 + a_8 V + a_{11} W + 0.5 a_{13}\zeta + a_{14}\xi \quad (6c)$$

$$A_{21} = b_3 + b_7 V + b_{10} W + b_{12}\zeta + 0.5 b_{13}\xi \quad (6d)$$

$$A_{22} = b_4 + b_8 V + B_{11} W + 0.5 b_{13}\zeta + b_{14}\xi \quad (6e)$$

These computations are suitable for use on a minicomputer. The number of input variables are small as shown by this list.

2 nominal launch angles
2 nominal burnout angle changes (if required)
1 no-wind azimuth (if required for variable launch directions)
28 polynomial coefficients
33 total number of inputs to real-time program.

Implementation of the Method

A small computer program, named BALWINCO (BAListic WINd COmpensation), was developed to implement the coordinate transformation and curve-fitting procedures required by this method. A copy of the computer program and the data used in this analysis are available from the author. The program performs essentially three different functions:

1) Generates the wind and launcher angle conditions for the 24 runs. Punched card output is prepared for direct insertion into the nominal trajectory simulation input deck.

2) Accepts the results of the simulation output, transforms to the $\zeta-\xi$ system and curve-fits these data for polynomial coefficients. Prints tables of launcher settings vs ballistic wind field components.

3) Accepts a specific ballistic wind vector and determines the launcher angles. Also prints the partials of the burnout $Z-\Xi$ angles with respect to the launch $\zeta-\xi$ angles for input to an iterative 6-DOF simulation.

Both the input and punched card output nomenclature and format were made compatible with the trajectory simulation program. So the cards punched by either program can be read directly by the other program. This greatly reduces the chance for error, since manual transcription and repunching of the data are not required. It should be noted that the simulation program had been previously modified to use geodetic tangent plane referenced angles for input of vehicle attitude and initial velocity. The wind vector, however, is referenced to a geocentric tangent plane. This slight discrepancy was felt to not materially degrade the results.

Table 3 Dimensionless values for curve-fitting process

W_x	W_y	ζ	ξ	Z	Ξ
−0.5	−0.5	−0.5	−0.5	−0.01	−3.19
−0.5	−0.5	−0.5	0.5	0.05	−0.10
−0.5	−0.5	0.5	−0.5	3.03	−3.34
−0.5	−0.5	0.5	0.5	3.29	−0.27
−0.5	0.5	−0.5	−0.5	−3.23	−3.15
−0.5	0.5	−0.5	0.5	−3.35	−0.04
−0.5	0.5	0.5	−0.5	−0.18	−3.14
−0.5	0.5	0.5	0.5	−0.11	−0.05
0.5	−0.5	−0.5	−0.5	0.10	0.02
0.5	−0.5	−0.5	0.5	0.20	3.38
0.5	−0.5	0.5	−0.5	3.43	−0.15
0.5	−0.5	0.5	0.5	3.72	3.19
0.5	0.5	−0.5	−0.5	−3.38	0.10
0.5	0.5	−0.5	0.5	−3.46	3.48
0.5	0.5	0.5	−0.5	−0.05	0.09
0.5	0.5	0.5	0.5	0.07	3.45
1.0				0.16	3.49
−1.0				−0.12	−3.22
	1.0			−3.44	0.07
	−1.0			3.44	−0.23
		1.0		3.28	−0.18
		−1.0		−3.28	0.01
			1.0	0.10	3.33
			−1.0	−0.07	−3.12

It is essential when dealing with such disparate quantities as wind components and attitude angle changes in the same matrices to scale one or both sets of quantities. Both variables were nondimensionalized by selecting a wind unit and an angular unit which, when applied to the nominal trajectory, resulted in approximately equal burnout perturbations. This consideration fixes the ratio between the units. The magnitude was chosen to include a large amount of the expected operability range of the rocket launch system. These considerations resulted in selecting the wind unit as 40 ft/s and the angular unit as 5.5 deg based on the previous campaign's wind weighting analysis. All wind and angle quantities are divided by these units to yield dimensionless values for use in the curve-fitting process.

The wind perturbations were defined as head- and crosswind perturbations relative to the perturbed azimuth. This was found to produce larger than expected curve-fitting errors. Rotating the winds so that they were defined as head- and crosswind components relative to the nominal azimuth reduced the curve fit 1σ errors to 0.026 units for both Z and Ξ (about 0.144 deg). This meant only that the regular four-dimensional pattern of perturbed runs was not really

Table 4 Z curve-fitting polynomial coefficients

W_x	W_y	ζ	ξ	Type of term
0.14149141	−3.4387178	3.2823057	8.61203671E-02	Linear
2.16379613E-02	−0.26272696	0.28345519	3.60540561E-02	Second-order in W_x
	−3.67460609E-03	9.36423615E-03	−0.18395358	Second-order in W_y
		−3.98834934E-04	0.19497329	Second-order in ζ
			1.48032419E-02	Second-order in ξ

Table 5 Ξ curve-fitting polynomial coefficients

W_x	W_y	ζ	ξ	Type of term
3.3558846	0.14950532	−9.28285718F-02	3.2259283	Linear
0.13115048	3.89560722E-02	−2.41529644F-02	0.27285421	Second-order in W_x
	−8.07268023E-02	0.15759993	1.76623352E-02	Second-order in W_y
		−8.76964331E-02	−1.79814324E-02	Second-order in ζ
			0.10459912	Second-order in ξ

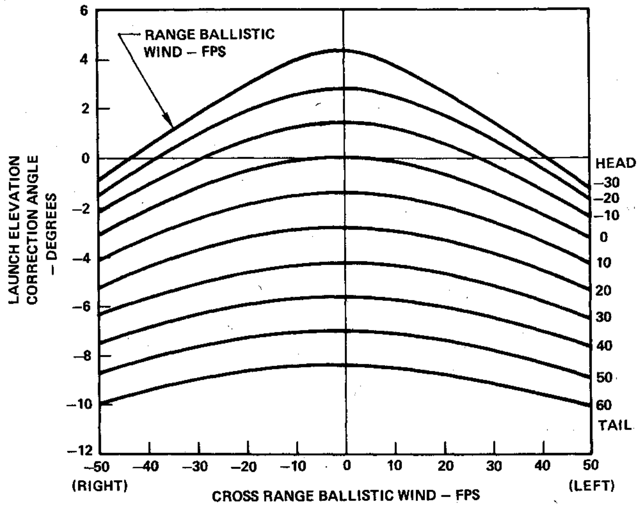

Fig. 3 Launcher elevation angle correction.

produced, but that the irregular pattern was certainly adequate. The determinant of the Jacobian was reduced from the regular 24-hedroid value of 62,208 to 37,932.

Evaluating the polynomials on the regular 24-hedroid vertices defined with the wind axes referenced to the nominal azimuth produces the results which would have been obtained. Table 3 gives the dimensionless values for the curve-fitting process. The first four columns give the ordinates of the vertices of a regular 24-hedroid. The first 16 points are the vertices of a hypercube. The last 8 points are the remaining vertices of the 24-hedroid. For clarity only the nonzero components of the rows describing the vertices are printed. These 24 perturbations of ballistic wind components and launch angles were input to the 6-DOF simulation. The last two columns are the simulation output in terms of Z and Ξ angles at burnout. These are also nondimensional.

The curve fit coefficients for ζ and ξ polynomials are shown in Tables 4 and 5. Again, the data are nondimensional. The format of these tables shows the four linear coefficients on one line. The second-order terms are shown as an upper triangular square matrix on the next four lines. The lower triangular portion of the matrix is all zeroes and, for clarity, is not shown.

Solving the polynomials for a matrix of winds yields elevation and azimuth launcher setting corrections are functions of range and cross-range wind. These are shown in Figs. 3 and 4. These results are in units of degrees and are a graphical representation of the firing tables for the winter 1976-1977 launches at Wallops Island. These plots are carried out to 50 ft/s crosswind and from 30 ft/s headwind to 60 ft/s tailwind. Any larger headwinds require an "over-the-shoulder" launch which, while technically sound, is not politically viable in the Wallops area.

Test Case Results

Six test case ballistic winds were tried against the new method of wind compensation. Four of these cases were real-

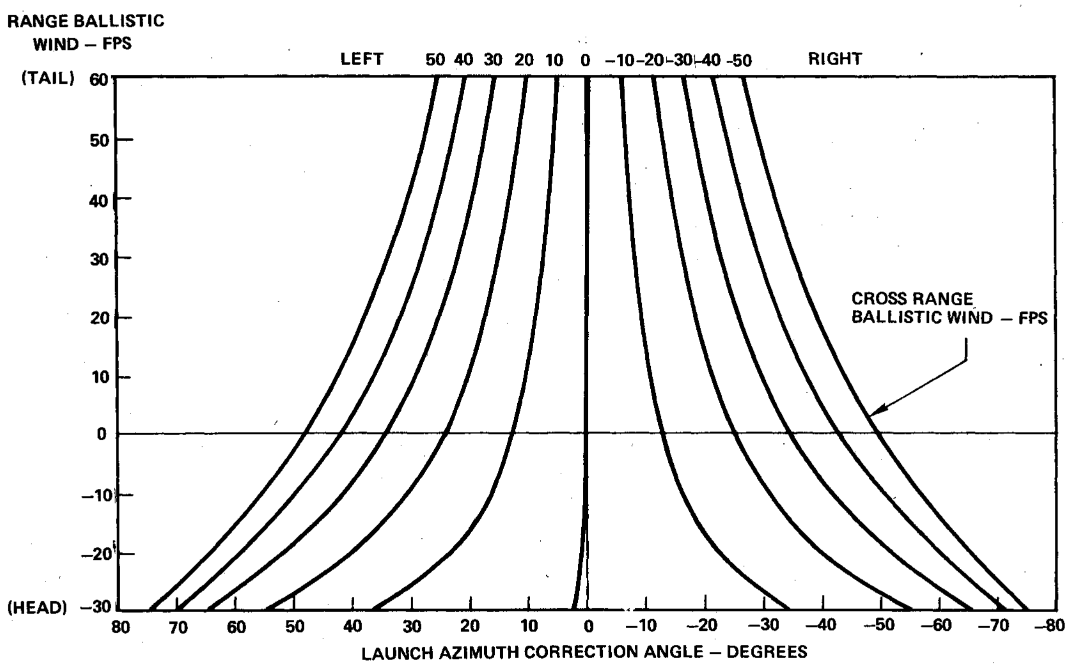

Fig. 4 Launcher azimuth angle correction.

Table 6 Launcher setting table test cases

Designation	Tailwind, ft/s	Left crosswind, ft/s	θ, deg	ψ, deg	$\Delta\gamma$, deg	$\Delta\sigma$, deg
Nominal	0	0	83.700	140.000	0	0
3/16/76	−14.1	−54.0	81.274	77.958	−0.04169	0.8853
3/31/76	−29.8	−21.8	86.394	82.435	0.07268	0.3701
Wind C	5.70	−21.59	82.285	116.147	−0.01060	0.0487
Wind D	−3.41	−13.32	83.916	121.742	0.00350	0.0502
Test #1	22.5	12.5	80.314	150.410	−0.01607	−0.0034
Test #2	12.0	46.0	79.563	179.247	−0.03294	−0.2401
				Means	−0.00549	0.18513
				1 σ	0.03740	0.36001
				rms	0.03761	0.40482

wind fields which had been troublesome on previous launch attempts. The last two were somewhat arbitrary choices to test particular quadrants of the wind vector. The nominal values and displaced launcher settings are shown in Table 6. Each of these cases was simulated and the discrepancy in burnout angles from the nominal determined. The statistics of these data are shown in Table 6. Considering the magnitude of the winds involved, the observed errors are quite acceptable.

Having run these test cases, it was decided to add the results into the data for the curve fitting. This altered the launcher setting curves a very small amount, as would be expected. The determinant of the Jacobian increased to 3,320,080. The resulting 30-point data set was the basis of the launch support for two Athenas at Wallops. Both vehicles were successful. Due to operational considerations, the launcher setting is based on wind data many minutes old. Since the mission of the Athena at Wallops is to probe nearby storms, it is very difficult to make any statement about the experienced accuracy of the new method. Storm conditions around the launch site, coupled with the age of the measurements, make precise wind compensation impossible. The simulated accuracy comparison presented before is more meaningful than actual flight data comparison.

The previous method of generating the launch setting tables required over 50 iterated 5-DOF trajectories. About 15 iterated 6-DOF trajectories would have been required to provide corrective information. The trends shown by cross-plotting the 5-DOF data would have been graphically shifted to incorporate the 6-DOF solutions. Graphical interpolation would then be performed to complete the launcher setting tables. The computer time alone would have cost about twice that used by the new method. Time-consuming graphical manipulation of the data was eliminated by the use of curve fits in the W_x-W_y-ζ-ξ space. As demonstrated by the test case results, the reduction in direct cost was matched by an increase in accuracy over the similar test cases generated during the previous campaign's software development.

Method of Simplification

Since it was decided to use only a regular pattern of points, the smallest number of data points required was 24. However, only 14 coefficients of the polynomials need to be evaluated. So it would seem than an irregular pattern of points could be found that would use, say 14 or more perturbed trajectories to generate the data required for a complete high-order ballistic wind compensation analysis. An optimum pattern of such points can be found by numerical second-order search techniques. Such a technique has been used to determine that set of points on a unit hypersphere which produce the maximum value for the determinant of the Jacobian matrix. This criterion for optimality is motivated by the requirement that the Jacobian matrix not be singular. So it seems that an optimal set of sampling points would maximize the determinant of the Jacobian.

Optimal hypersolids of 14 through 23 points have been determined by a systematic relaxation procedure starting with the 24-hedroid. These solids provide a nonsingular solution to the curve-fit coefficients. This analysis was performed after the wind compensation analysis and so was not used for launch support.

Conclusions

A ballistic wind compensation method has been presented that uses noniterative trajectory runs. The data from these runs are transformed to a nearly linear-response system and fit to second-order polynomials. These polynomials are then used in an iterative scheme to solve for the launcher angles for any specific wind field. All the computations, other than the trajectory simulations, are suitable for minicomputers and moderately complete calculators. The variable launch azimuth feature is easily incorporated, as is the capability to do some trajectory changes as part of the wind compensation. The method is quite accurate even with high winds.

Only a small number of trajectory simulations need to be performed to yield all the required information. So the method achieves both high accuracy and low cost.

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